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Research Article

A Study on Digital Analysis of Bach's "Two-Part Inventions"

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In the field of music composition, creating polyphony is relatively one of the most difficult parts. Among them, the basis of multivoice polyphonic composition is two-part counterpoint. The main purpose of this paper is, through the computer technology, conducting a series of studies on "Two-Part Inventions" of Bach, a Baroque polyphony master. Based on digitalization, visualization and mathematical methods, data mining algorithm has been applied to identify bipartite characteristics and rules of counterpoint polyphony. We hope that the conclusions drawn from the article could be applied to the digital creation of polyphony.

1. Introduction

1.1. Patterns. In the process of composition, the composer will always follow inspirations and then proceed according to a certain mode. Various types of patterns and rules are available in music works, the audience's understandings on music can be expressed in a formalized way through a series of rules [1, 2]. These rules enables the audience to generate hearing expectation, which exists in different dimensions such as melody, rhythm, and harmony, and produces different patterns and pieces on the basis of constant changes in basic elements [1, 2]. Fractal geometry originated in the nineteenth century. Fractal sets are the geometry of chaos. It is an important branch in modern mathematics. Some famous mathematicians discovered the existence of a special structure and morphology with study on continuous nondifferentiable curves.

1.2. Our Works. This paper applied the MATLAB tool to visualize MIDI music data and observed and unveiled pattern features of polyphonic music with intuitive techniques. We emphasized our analysis on the No. 1 to No. 5, No. 6, No. 8, No. 13, and No. 14 of Two-Part Inventions (Johann Sebastian Bach). Experimental analyses were made in terms of the pitch and the tone, and the application of these patterns and rules in computerized digital music composition was discussed in the end.

2. Experiments

2.1. Preparations. The pitch is a very important element in the music; the audiences are very sensitive to changes in the pitch, and they are capable of feeling only 0.5% of change [1, 2]. Constant change in the pitch is reflected in the process of music, and the interval of change between pitches is very important in Western music system. We retrieved information from a MIDI file [3] and only utilized partial information in order to simplify the process. We defined a matrix **M** as

$$\mathbf{M} = \{D_i \;\; S_i \;\; P_i \;\; T_i\}, \qquad (1)$$

where S_i is the start tempo, D_i is continuing tempo, P_i is the pitch, and T_i is the pitch interval; that is,

$$T_i = P_{i+1} - P_i. \qquad (2)$$

2.2. Pitch Intervals and Statistics. This paper conducted statistics and analysis on semitone spaces of adjacent pitches in Bach's works, and results were shown in Table 1. It is clearly

Table 1: Statistics of semitone spaces of adjacent pitches in Bach's inventions (intervals bigger than 12 ignored); unit: %.

Works	Intervals											
	1	2	3	4	5	6	7	8	9	10	11	12
No. 14	0.003	0.177	0.358	0.119	0.058	0.07	0.034	0.068	0.034	0.034	0.014	0.003
No. 13	0.007	0.09	0.08	0.368	0.14	0.067	0.054	0.107	0.033	0.043	0.003	0.003
No. 8	0.003	0.2	0.278	0.119	0.078	0.034	0.027	0.06	0.054	0.047	0.04	0.003
No. 6	0.013	0.294	0.343	0.125	0.086	0.043	0.013	0.013	0.013	0.017	0.01	0.003
No. 5	0.005	0.26	0.455	0.13	0.059	0.026	0.005	0.003	0.01	0.02	0.008	0.015
No. 4	0.025	0.249	0.5	0.036	0.018	0.025	0.007	0.029	0.014	0.04	0.043	0.011
No. 3	0.036	0.266	0.409	0.084	0.033	0.055	0.026	0.007	0.018	0.026	0.036	0.004
No. 2	0.039	0.296	0.4	0.056	0.028	0.09	0.01	0.014	0.017	0.023	0.014	0.011
No. 1	0.016	0.234	0.413	0.163	0.07	0.028	0.004	0.02	0.016	0.01	0.016	0.008

seen that 1, 2, and 3 semitones represent the largest proportion of semitone spaces in the nine works of Bach under study. Effects of melodic interval and harmonic interval are similar, and they arouse different psychological feelings, like harmonic interval does, and let people have expectations, thus developing continuously from music thoughts. We can define a simple rule for composition in accordance with the statistics:

> *when a music event sequence N is given, the proportion of semitone space between N_{i+1} and N_i at 1, 2, and 3 should be greater than 70% (Rule I) in the development of N (Rule I).*

2.3. Melody Intervals. In addition to overall statistics on this type of interval spaces, we also want to understand specific laws of changes in pitch interval during the marching process of the melody. We defined a set **S**, each element of **S** is a data pair $\langle E_i \quad E_j \rangle$, representing that the pitch interval T_i between MIDI note event i and $i+1$ on a voice part P_1 is equal to pitch interval T_i' between P_2 MIDI note event j and $j+1$ on the other voice part; namely, $T_i = T_i'$. We mapped the pitch interval with MATLAB drawing tool, and the results are shown in Figure 1.

The following characteristics can be concluded by analyzing the above diagram: continuous circle lines occur in the linear system (slope: 1), a small starting piece repeatedly occurs at different positions in the second voice part. We can see, according to transitive relation, that lots of repetitive pieces occur on single parts, and by mapping the distribution diagram we can also find out that such pieces are occurring repeatedly in one voice part.

With further analysis of the music data, we know that the piece in the slope of Figure 1 maps to the note event as shown in Table 2. What is interesting is that the repeated piece is imitated differently. Some are imitated partly, while some fully. As shown in the original music staff in Figure 2, we can see that a piece of different event notes follows the repeated piece. They have different ostinato which gives the listener a sense of change.

It can be found from Table 3 that repeated interval pieces represent a very large proportion in Bach's two-part inventions, and most of them (except No. 4) are at least 50%.

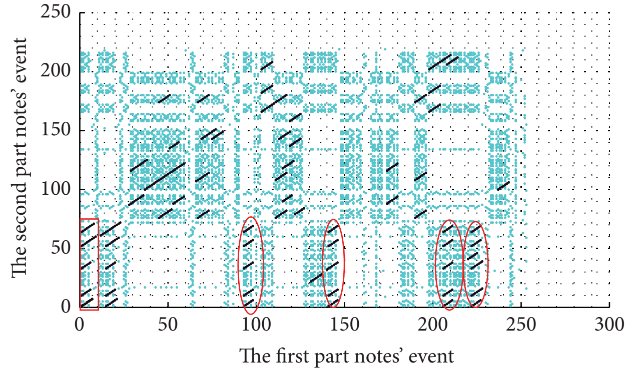

Figure 1: Equal pitch interval distribution diagram on two voice parts of Bach's inventions No. 1. Horizontal and vertical ordinates represent the two voice parts' MIDI note event sequence numbers; the small circles in the coordinates mean that pitch intervals of two voice parts are the same.

Table 2: Repeated piece sequence. Each line is, respectively, the start note and end note event number of first voice part and the start note and end note event number of the second.

First part start note	First part end note	Second part start note	Second part end note
1	6	10	15
1	6	33	38
1	7	1	7
1	8	64	71
1	9	52	60

During further analyses, we were aware that such repeated pieces occurred at different starting points of the pitch, and such repeat occurred at different tonalities. Therefore, we can define a new rule:

> *when given a particular pitch interval sequence M to form a complete repertoire of music event sequence N, sequence $P_1, \ldots, P_k$ can be used to make the pitch interval sequence M repeat k times in N, with P_1 being the starting pitch every time (Rule II).*

Table 3: Total proportion of note events in repeated piece in Bach's Two-Part Inventions. *First* represents the first voice part, and *Second* stands for the second voice part; unit: %.

Part	Works								
	No. 1	No. 2	No. 3	No. 4	No. 5	No. 6	No. 8	No. 13	No. 14
First	56.39	83.38	46.59	28.10	90.24	49.06	59.73	41.92	52.36
Second	52.37	78.77	56.16	28.22	86.06	46.12	61.10	31.95	51.36

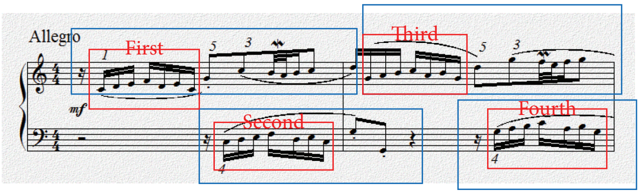

Figure 2: The original music staff piece. The blue slope refers to the music sentence, and the red refers to repeated music piece.

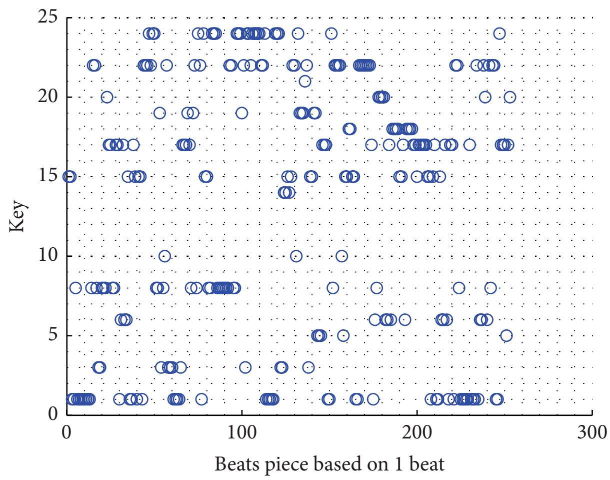

Figure 3: Tonality distribution diagram of Bach's Two-Part Inventions No. 1 based on 1 beat per piece (when $b = 1$). Vertical axis represents the tonality; 1 to 12 correspond to majors extending from Major B to Major C, and 13 to 24 correspond to minors extending from Minor C to Minor B.

2.4. Tonality. As regards researches on tonality, a great many researchers have put forward plenty of models to describe changes in the tonality. Krumhansl proposed an algorithm to measure the music data and to determine perceivable tonality [1, 2, 4] on the basis of relevance with the attribute data of major and minor tonality measured by experience. Krumhansl's algorithm is called K-Finding algorithm which is used to find out the main tonality of a piece of music. The method has shown great accuracy in measuring classical music such as Bach's works. In our experiment, we apply Krumhansl's K-Finding algorithm to analyze the change law of tonal characteristics of creative music in Bach's inventions which we are going to study.

$\mathbf{M}$ signifies the matrix of simplified music event; minimum k was calculated from the first note event $\{S_1\ \ D_1\ \ P_1\ \ T_1\}$ in sequence, making the total duration from the first note event to the kth note event $D = d_1 + d_2 + \cdots + d_k$, $D > b$ tempos (b is user defined value). Then we applied the K-Finding algorithm to analyze the tonality key of $\mathbf{M}_1 \cdots \mathbf{M}_k$ in this piece and repeated the above-mentioned process with the second note event in $\mathbf{M}$ as the first music note to get the second piece, until all note events were measured; in this way, we could obtain a set of music piece tonal change data. We used a visual method to map out the data, as shown

Table 4: Rising and falling keys of various tonalities (Major).

Major	Rising or Falling
Major C	
Major F	Falling B
Falling Major B	Falling B, Falling E
Falling Major E	Falling B, Falling E, and Falling A
Falling Major A	Falling B, Falling E, Falling A, and Falling D
Falling Major D	Falling B, Falling E, L Falling A, Falling D, and Falling G
Falling Major G	Falling B, Falling E, Falling A, Falling D, Falling G, and Falling C
Major B	Rising F, Rising C, Rising G, Rising D, and Rising A
Major E	Rising F, Rising C, Rising G, and Rising D
Major A	Rising F, Rising C, and Rising G
Major D	Rising F, Rising C
Major G	Rising F

Table 5: Rising and falling keys of various tonalities (Minor).

Relative minor	Rising or Falling
Minor A	Rising G
Minor D	Falling B, Rising C
Minor G	Falling B, Falling E, and Rising F
Minor C	Falling E, Falling A
Minor F	Falling B, Falling A, and Falling D
Falling Minor B	Falling B, Falling E, Falling D, and Falling G
Falling Minor E	Falling B, Falling E, Falling A, Falling G, and Falling C
Rising Minor G	Rising C, Rising G, Rising D, Rising A, and Heavy Rising F
Rising Minor C	Rising F, Rising C, Rising G, Rising D, and Rising B
Rising Minor F	Rising F, Rising C, Rising G, and Rising E
Minor B	Rising F, Rising C, and Rising A
Minor E	Rising F, Rising D

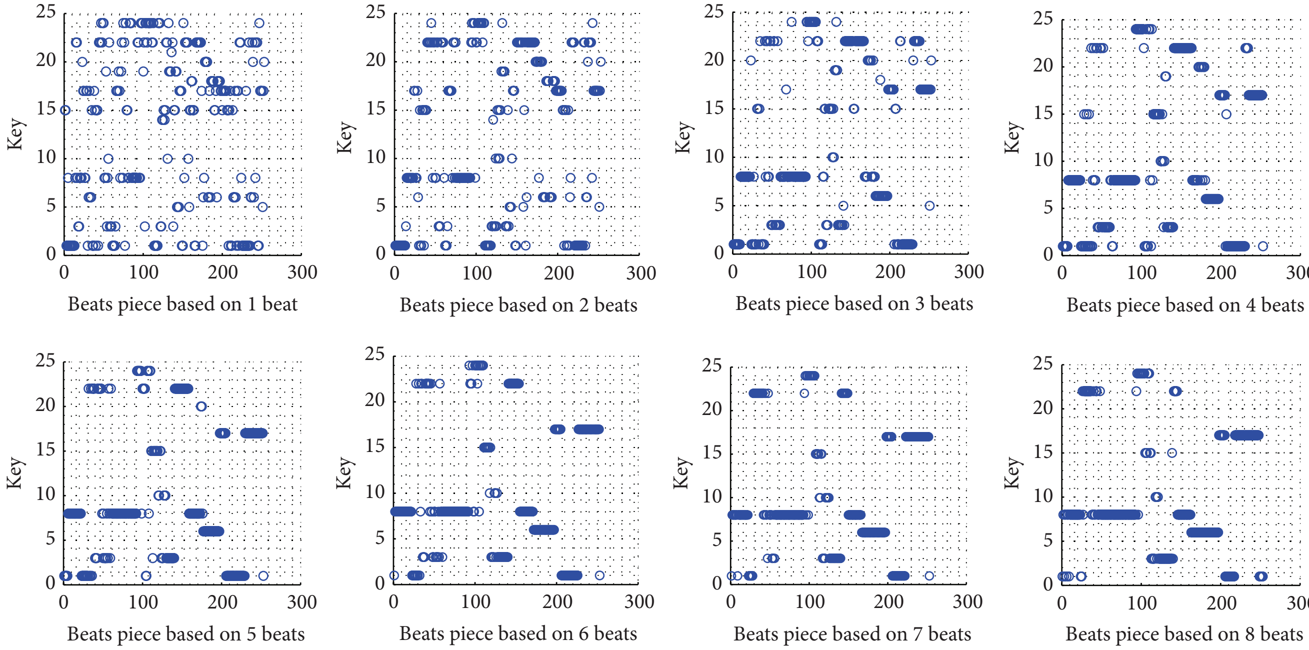

FIGURE 4: Summary of tonality distribution diagram for Bach's Two-Part Inventions No. 1 (when $b = 1$–8).

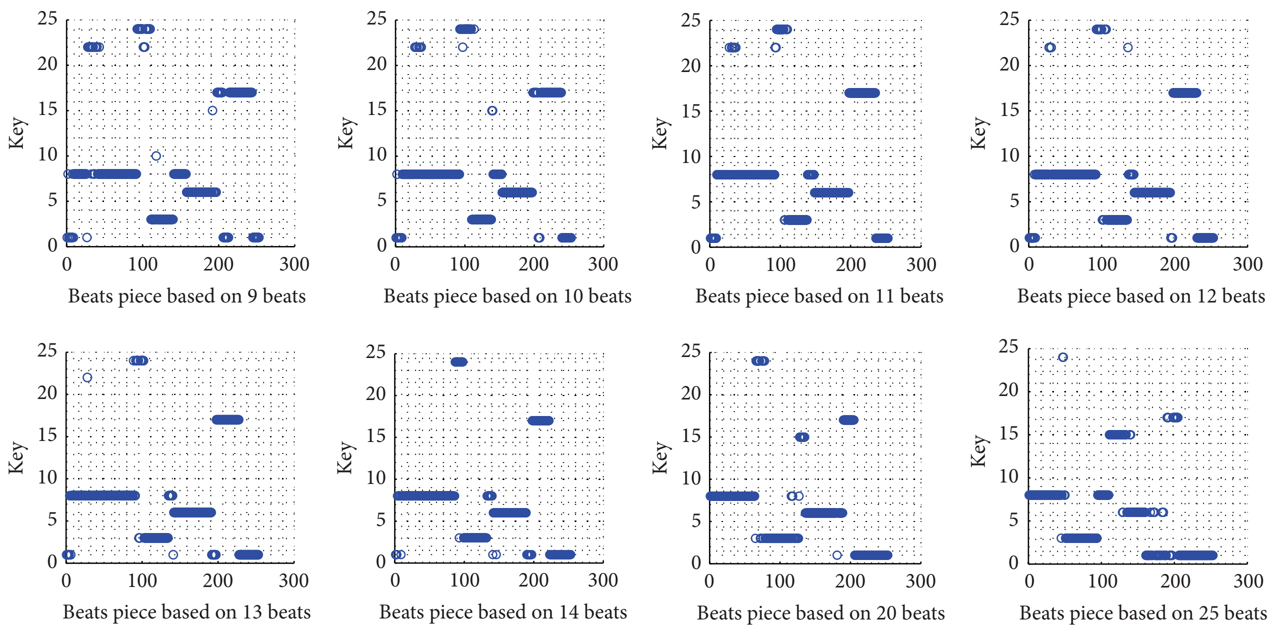

FIGURE 5: Summary of tonality distribution diagram for Bach's Two-Part Inventions No. 1 (when $b = 9$–25).

in Figures 3, 4, 5, and 6: data distribution of Bach's Two-Part Inventions No. 1.

It can be concluded that melodies converge on several different tonalities when rhythm lengths of benchmark pieces are different. According to the data collected, although Invention No. 1 is a Major C piece, its piece tonality is constantly changing in relation to the tonality of the whole work.

In order to further analyze the change rule of the tonality, we use Tables 4 and 5 to illustrate the relationship between changing pieces and the tonality.

We used the piece tonality distribution diagram to analyze the characteristics of a music piece's changes in tonality under the Krumhansl model, and the results are shown in Table 6.

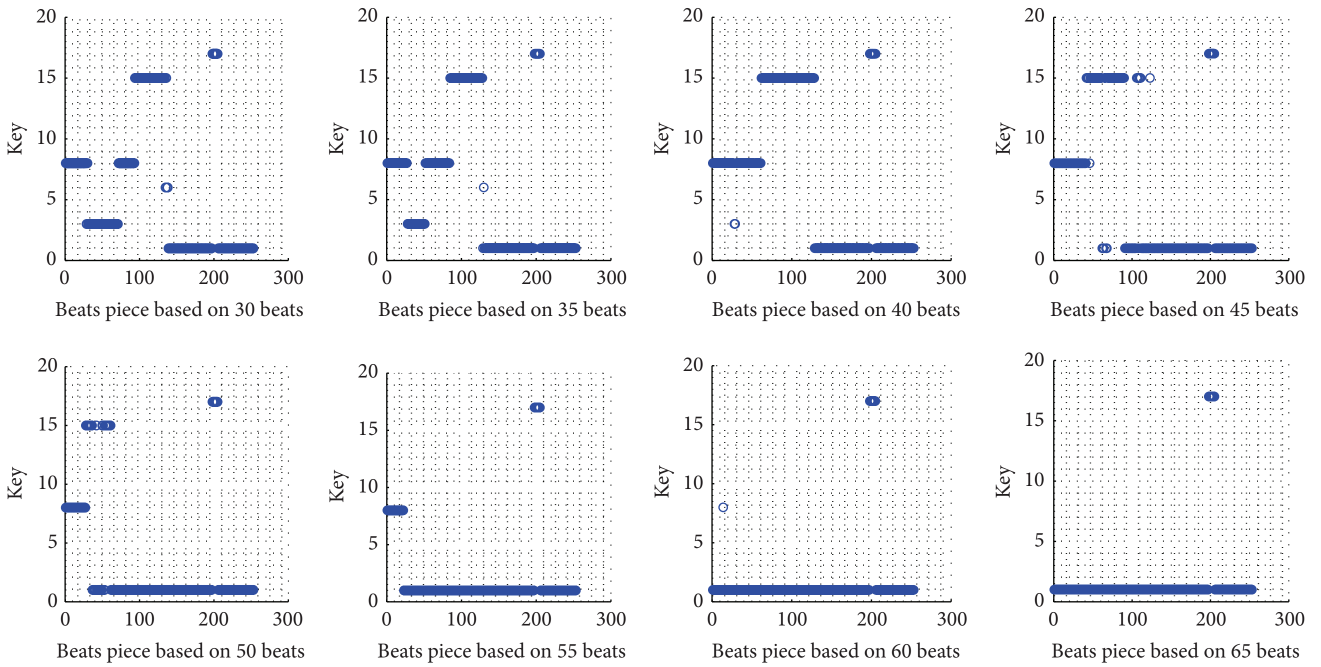

FIGURE 6: Summary of tonality distribution diagram for Bach's Two-Part Inventions No. 1 (when $b = 30$–65).

TABLE 6: Tonality changes in pieces of Bach's Two-Part Inventions (based on Krumhansl's model). The spaces listed in the third column are minimum interval spaces of the piece's tonalities after the major and relative minor formed loops in which the relative minor and major connect together and the first column connects to last column in Tables 4 and 5.

Works	Main tonalities	Spaces
Number 1	Major C, Major G, Minor D, and Minor E	1
Number 2	Minor G, Rising Major A, Minor c, and Major C	2
Number 3	Major A, Minor B, and Major D	1
Number 4	Minor D, Minor A, and Major C	2
Number 5	Rising Major D, Minor F, and Rising Major G	1
Number 6	Major B, Major E	1
Number 8	Major F, Major C, Minor D, and Major A	1
Number 13	Minor E, Major C, and Minor A	1
Number 14	Rising Major F, Rising Major A, and Major D	2

It can be concluded from Table 6 that whenever there are tonality changes, normally a tonality with minimum rising or falling values adjacent to the given main tonality will be selected for change purpose. In line with the above analyses, we can develop a new rule:

> *when composing a complete music event sequence N, N may consist of k music sequences, and when the piece tonality under the Krumhansl model is no more than 12 ($b \geqslant 12$), the space between tonalities of k music sequences should be no more than 2 (Rule III).*

This rule is of great significance, and in the case of connecting repeated pieces, this method of tonality change may be used to analyze possibly connected pieces.

3. Discussion

In our experiment, we concluded the patterns and rules in Bach's Two-Part Inventions, which is typical of polyphony works, and we discovered three characteristic rules (*Rules I–III*) in Bach's Two-Part Inventions. Nevertheless, it needs pointing out that these three rules only cover the pitch and the tonality, with no consideration for the rhythm, melody, and harmony. Studies show that global context has an effect on music perception [5]. William did a lot a lot of experiments to study the effects on music perception of the integration of pitch and rhythm. Results show that the integration of the individual music parameter cannot be combined easily. They have effects on each other after integration [6]. So, the modeling of music is difficult; we need to study it further rather than applying the three rules everywhere.

4. Conclusion

Computerized musical composition includes auxiliary composition, algorithm composition, and works' compilation. The three rules we put forward are applicable for basic rules of polyphony works with styles similar to Bach's; in computerized algorithm composition these three rules can be used for assessing and selecting works with better polyphony styles, and they can be used in auxiliary computer composition to inspire the composer with musical pieces generated from these rules so as to speed up the efficiency of composition. In addition, these three rules can be used for identifying the characteristics of existent works and for categorizing various types of works.

Conflict of Interests

The authors declare that they have no competing interests regarding the publication of this paper.

Authors' Contribution

All authors completed the paper together. All authors read and approved the final paper.

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